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Analytical study of the combustion waves propagation under filtration of methane–air mixture in a packed bed

V.I. Bubnovich [a], S.A. Zhdanok [b], K.V. Dobrego [b,*]

[a] Department of Chemical Engineering, Universidad de Santiago de Chile, Santiago, Chile
[b] Chemical Physics Laboratory, Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, P.Brovki Street 15, Minsk 220072, Belarus

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Abstract

The study of combustion waves during the filtration of lean methane–air mixtures in inert porous media is carried out using the one-temperature approximation in a semi-infinite canal. The analytical solution is built in three different regions, the pre-heating region, the reaction region and the region occupied by the combustion products. By means of the solution, the temperature and mass fraction profiles of the methane are built for three regions, as well as the longitudinal extension of the reaction region, the ignition temperature of the mixture and the combustion wave propagation velocity in the system. The results obtained are validated by means of comparisons with the known numerical and analytical solutions and experimental results found in literature. Finally, an analysis of the built analytical solution in terms of five dimensionless parameters is made, which define heat and mass transport in mathematical model.

Keywords: Combustion; Inert porous media; Analytical solution

1. Introduction

Recently, the combustion of gas in porous media has been the focus of numerous researchers who work in the areas of combustion and the environment due to its interesting industrial applications [1–4] such as: oil extraction, infrared burners and heater development, ceramic materials synthesis, porous catalysts, grounds polluted by toxic organic shedding recuperation, destruction of volatile organic compounds (VOC) in air, hydrogen production, diesel engines, and pollution control. The process by which the region of exothermic chemical reactions propagates along inert porous media must be viewed within the framework of combustion waves in these types of media as presented schematically in Fig. 1.

It is known from the literature [5–7] that during gas mixture combustion in inert porous media, combustion waves that move up-stream or down-stream along the system can be observed. The direction of these movements depends generally on the physical properties of both solid and gas as well as the initial speed, temperature and excess air of the mixture. Conjugating these parameters, movement speeds of said waves are achieved that are much lower to those of gas and the temperature profiles showing a very pronounced maximum in the reaction region.

Due to the active participation in this process of both the porous media and the reacting gas, three characteristic regions can be identified inside the porous media. There is a region in front of the combustion region, where reacting gases are mixed in naturally and pre-heated using heat lost by the porous media. At the entrance of this region, the temperature, the gas speed and the equivalence ratio are controlled. At the same region of the porous media, $0 < z < l_1$, the gaseous mixture is inflamed as a result of a pre-heating of a the porous media zone of several centimetres in length by an external heat source (for example, an electrical resistance). The second region is a luminous tight

* Corresponding author. Tel.: +375 17 2842217; fax: +375 17 2842212.
E-mail address: kdob@itmo.by (K.V. Dobrego).

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Nomenclature

a	heat transfer coefficient [W/(m^2 K)]
C_p	specific heat [J/(kg K)]
d	diameter of glass tube [m]
d_p	diameter of the solid alumina spheres [m]
D	molecular diffusion coefficient [m^2/s]
E/R	activation energy [K]
h	convection heat transfer coefficient [W/(m^2 K)]
h_c	heat of chemical reaction [J/kg]
K	pre-exponential factor [s^{-1}]
T	temperature [K]
t	time [s]
u_g	average interstitial gas velocity [m/s]
u_{FC}	combustion wave velocity [m/s]
w	mass fraction of species, dimensionless
x	spatial distance in moving coordinates [m]
$\bar{x}$	dimensionless spatial distance
z	spatial distance [m]

Greek symbols

β_v	effective coefficient for heat exchange with surroundings [W/(m^3 K)]
ε	porosity, dimensionless
ε'	emissivity, dimensionless
ε''	transmissivity factor, dimensionless
φ	coefficient of excess air, dimensionless
λ	thermal conductivity [W/(m K)]
θ	dimensionless temperature
ρ	density [kg/m^3]
σ	Stephan–Boltzmann constant

Subscripts

g	gas
s	solid
0	inlet

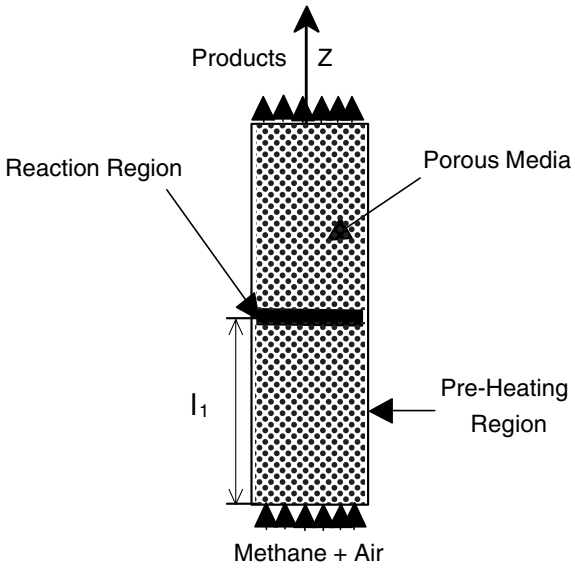

Fig. 1. Scheme of the considered physical problem.

one, and represents the chemical reaction region which moves in the same direction or against the gas flow, and it is here where a huge amount of enthalpy is absorbed by the porous media and directed to the first region where the fresh mixture enters. Because of the big specific surface of the porous media, this last region delivers energy to the incoming gas mixture which is transferred by convection to the reaction region. Thus, heat regeneration occurs which implies an enthalpy excess at the chemical reaction region and a partial increase of the front temperature, which can exceed the adiabatic temperature [8,9]. Moreover, the mass transport by diffusion and the heat transport by thermal radiation are considerable at this region. Finally there is a third region which is placed forward of the front, and has combustion products, which actively exchanges convection heat with the porous media leaving the system at almost ambient temperature. These systems are characterized by the presence of two dynamic fronts, one is the chemical reaction front and the other is the high temperature front. As a result of the adequate selection of geometric properties, dynamic flows and thermal parameters, the two fronts can superimpose and intensify reciprocally.

Today, gas combustion in inert porous media is intensively studied. There are numerical [10,11], analytical [12,13] and experimental studies [9,14,15] of the topic, where relations are established like formulas or graphics between major properties of a combustion wave as the highest temperature, the speed and the movement direction of the wave and biphasic system physical properties and conditions at system entrance. In spite of this, there are relatively few studies of the mixture ignition temperature in the biphasic system and the chemical reaction region thickness. Normally, to inflame a mixture in the numerical studies [16], a temperature not inferior to the required is imposed on a region of several centimetres of extension in the inert porous media. However, in practice, it is very important to know the precise value of this temperature, because higher temperatures compromise solid materials and originate toxic gases. In analytical solutions, combustion region thickness is normally ignored in order to be able to integrate the energy equation, in which the right side has an extremely non-lineal term.

Akkutlu and Yortos [17] study the properties of forward combustion fronts propagating at a constant velocity in the presence of heat losses. Heat losses are assumed to be relatively weak and they are expressed using two models: (1) a convective type, using an overall heat transfer coefficient; and (2) a conductive type, for heat transfer by transverse conduction to infinitely large surrounding

formations. Like result, they develop expressions for temperature and concentration profiles and the velocity of the combustion front, under both adiabatic and non-adiabatic conditions, in analytical form. An explicit expression is also obtained for the effective heat transfer coefficient in terms of the reservoir thickness and the front propagation speed. This coefficient is not only dependent on the thermal properties of the porous medium but also on the front dynamics.

Shkadinsky et al. [18] demonstrate the existence of both one and two stationary reaction zone structures which arise in filtration combustion in a moving porous medium. Using the narrow reaction zone approximation, they derive approximate analytical expressions for the principal combustion characteristics, including the combustion temperature, the temperature and depth of solid conversion at the first reaction zone, the locations of the reaction zones, the inlet and outlet oxidant fluxes, as well as profiles for the spatial distributions of pressure, temperature, and depth of conversion, corresponding to the stationary reaction zone structures.

Foutko et al. [12] present an analytical solution of this combustion wave assuming that the reaction speed in the combustion region is infinite and that the longitudinal extension of said region is null. Nevertheless, the construction of an analytical solution is here achieved without the help of those last two assumptions. It is known [19,20] that for bigger extension of the combustion region, the higher the production of NO_x at front and, consequently, it would be interesting to find out about the dependence of the thickness of that region on the rest of the physical parameters involved at the studied problem.

In this paper, the study of combustion waves during the filtration of lean methane–air mixtures in inert porous media is carried out using a one-temperature approximation in a semi-infinite canal. The analytical solution is built in three different regions: the pre-heating region, the reaction region and the region occupied by the combustion products, and requires one external entry parameter: the ignition temperature of the mixture. By means of the solution, the temperature and mass fraction profiles are built for the three regions, as well as the longitudinal extension of the reaction region and the combustion wave propagation velocity in the system. The results obtained are validated by means of comparisons with the known numerical and analytical solutions found in literature and with some experimental results. Additionally, based on Semenov thermal exposition analysis [21], an analytical development of mixture ignition temperature in inert porous media is made, and a parametric study is carried out of combustion wave properties as a function of dimensionless parameters defined at the mathematical model.

2. Basic equations and boundary conditions

The physical situation for which the mathematical model is built is represented in Fig. 1. It is assumed that the semi-infinite porous canal is made up of 5.6 mm diameter alumina spheres forming an $\varepsilon = 0.4$ porosity between particles. Other physical properties of the material are: specific heat $C_{p,s} = 1300\ \mathrm{J/(kg\ K)}$, mass density $\rho_s = 2500\ \mathrm{kg/m^3}$, and the thermal conductivity $\lambda_s = 1.3\ \mathrm{W/(m\ K)}$ which is considered to be large relative to that of the gas mixture. The heat transfer between the system and the environment occurs only through the porous media by natural convection and thermal radiation, and the effective coefficient for heat exchange with the surroundings, β_v, $\mathrm{W/(m^3K)}$, is found from the next energy balance applied to a cylindrical control volume of diameter D and dx long:

$$[h(T - T_0) + \varepsilon' \cdot \varepsilon'' \cdot \sigma \cdot (T^4 - T_0^4)] \cdot \mathrm{d}x \cdot \pi \cdot D = \beta_v \cdot (T - T_0) \cdot \mathrm{d}x \cdot \frac{\pi \cdot D^2}{4} \quad (1)$$

Then

$$\beta_v = \frac{4}{D}\left(h + \varepsilon' \cdot \sigma \cdot \varepsilon'' \frac{T^4 - T_0^4}{T - T_0}\right) \quad (2)$$

where $\varepsilon' = 0.45$ is the emissivity of the solid spheres, $\varepsilon'' = 0.38$ is a transmissivity factor for the tube material, $h = 10\ \mathrm{W/(m^2\ K)}$ is the heat transfer coefficient and σ is the Stefan–Boltzmann constant. The previously pre-mixed methane–air gas enters thorough section $z = 0$ with excess air equal to φ and an initial temperature T_0 and gas speed u_g. In addition, the mass density of the gas is $\rho_g = 1.13\ \mathrm{kg/m^3}$ and the specific heat is $C_{p,g} = 1100\ \mathrm{J/(kg\ K)}$. The flow is one-dimensional; the gas is incompressible and obeys the perfect gas law that assumes a local thermal equilibrium between the solid and the gas phases. For the species diffusivity, we assume that the molecular Lewis number is unity:

$$\frac{\lambda_g}{\rho_g \cdot C_{p,g} \cdot D} = 1 \quad (3)$$

It is assumed that the physical properties of both gas and solid are constant. In the initial part of the canal the mixture inflames and a narrow reaction region appears with a width equal to Δ which, after reaching section $z = l_1$, begins to move down-stream with a permanent speed of $u_{FC} \ll u_g$. The combustion of the mixture is described by means of the global chemical reaction in a single step:

$$CH_4 + 2(1 + \varphi)(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2\varphi O_2 + 7.52(1 + \varphi)N_2 \quad (4)$$

where φ represents the proportion of excess air in the reactant streams at the inlet to the system. The reaction rate is considered to obey the first-order Arrhenius equation with an activation energy of $E_a/R_0 = 15643.8\ \mathrm{K}$ and a pre-exponential factor $K = 2.6 \times 10^8\ \mathrm{s^{-1}}$. The entire region over the z-axis is divided into three regions: the gas pre-heating region, $0 \leqslant z \leqslant l_1$; the reaction region, $l_1 \leqslant z \leqslant l_1 + \Delta$; and the region occupied by the combustion products, $l_1 + \Delta < z \leqslant \infty$.

One of the purposes of the present study is the constructing of the temperature $T(t,z)$ and methane gas mass fraction $w(t,z)$ profiles as well as the reaction zone movement speed values u_{FC} and the width of the reaction region, Δ. As a result of these work assumptions, the mathematical model treated is

$$\frac{\partial w}{\partial t}+u_g\frac{\partial w}{\partial z}=\frac{\partial}{\partial z}\left(D\frac{\partial w}{\partial z}\right)-K\cdot w\cdot e^{-E/R\cdot T} \tag{5}$$

$$[(\rho C_p)_g+(\rho C_p)_s]\frac{\partial T}{\partial t}+u_g(\rho C_p)_g\frac{\partial T}{\partial z}=\frac{\partial\left(\lambda_s\frac{\partial T}{\partial z}\right)}{\partial z}-\beta_v(T_s-T_0)+h_c\cdot\rho\cdot w\cdot K\cdot e^{-E/R\cdot T} \tag{6}$$

The boundary conditions are

$$z=0:\quad T=T_0, w=w_0=1/(1+17.16(1+\varphi))$$
$$z\Rightarrow\infty:\quad T=T_0, w=0$$

3. Analytical method of solution

Before integrating the two differential equations (5) and (6), there is a shift to the reference system which moves together with the combustion wave: $x=z-u_{FC}\cdot t$, $u_{FC}\ll u_g$. Then it is assumed that the diffusion term in Eq. (1) is considerable only in the small reaction region: $u_g\cdot w'=-K\cdot w\cdot\exp(-E/R\cdot T)$. Substituting the last formula in the exponential term of Eq. (6), the mathematical model becomes

$$\overline{w}'=Le\cdot\overline{w}''-Fk\cdot\overline{w} \tag{7}$$
$$\alpha\cdot\Theta'=\Theta''-\beta\cdot\Theta-\gamma\cdot\overline{w}' \tag{8}$$

$$\overline{x}=\frac{x}{X_c},\quad \overline{w}=\frac{w}{w_0},\quad X_c=\frac{\lambda_s}{(\rho\cdot C_p)_g\cdot u_g}$$
$$Le=\frac{C_{p,g}\cdot D\cdot\rho_g}{\lambda_s}=\frac{\lambda_g}{\lambda_s},\quad Fk=\frac{K\cdot\lambda_s\cdot e^{-\frac{E}{R\cdot T}}}{\rho_g\cdot u_g^2 C_{p,g}}$$
$$\delta_1=\frac{l_1}{X_c},\quad \delta=\frac{\Delta}{X_c},\quad \Theta=\frac{T-T_0}{T_{ig}-T_0}, \tag{9}$$
$$\alpha=1-\frac{u_{FC}\cdot(\rho\cdot C_p)_s}{u_g\cdot(\rho\cdot C_p)_g},\quad \beta=\frac{\beta_v\cdot\lambda_s}{u_g^2\cdot(\rho\cdot C_p)_g^2}$$
$$\gamma=\frac{h_c\cdot w_0}{C_{p,g}\cdot(T_{ig}-T_0)}$$

In Eq. (7) coefficient Fk is a variable that depends on temperature. The temperature at each one of three regions will take a constant characteristic value, which allows the integration of Eq. (7). From the literature it is known [14] that combustion region thickness in a physical system like Fig. 1, is a few millimetres. Therefore, a characteristic distance called X_c is chosen, which for $u_g=0.43$ m/s and for the gas and solid physical properties already mentioned, has a value of 2.43 mm. The dimensionless boundary conditions are:

$$\overline{x}=-\delta_1:\quad \Theta_I=0,\overline{w}_I=1 \tag{10}$$
$$\overline{x}=0:\quad \Theta_I=\Theta_{II}=1,\Theta'_I=\Theta'_{II},\overline{w}_I=\overline{w}_{II} \tag{11}$$
$$\overline{x}=\delta:\quad \Theta_{II}=\Theta_{III},\Theta'_{II}=\Theta'_{III},\overline{w}=\xi \tag{12}$$
$$\overline{x}=+\infty:\quad \Theta=0,\overline{w}=0 \tag{13}$$

First, both equations are integrated in the pre-heating region. It is assumed that the diffusive term in Eq. (7) is very small, $Le\approx 0$, and the temperature present in the exponential term of Fk is constant and equal to the mean between the initial temperature of the mixture and the ignition temperature. In order to integrate the energy Eq. (8) in the same region, it is assumed that the chemical reaction is negligible, and $\overline{w}'=0$. Moreover, the first region extension is taken to be $l_1=25$ cm, for the following three reasons:

- Gas entrance section has to be away from combustion region, because the front cannot be affected by transient processes that occur near this section.
- In practice, this region is normally used to inflame the mixture.
- Some numerical value of l_1 is needed to be able to carry out numerical calculations with the purpose of validating the constructed analytic solution.

As a result of the integration of Eqs. (7) and (8) with the boundary conditions $\overline{w}=1$, $\Theta_I=0$ at $\overline{x}=-\delta_1$ and $\Theta_I=1$ at $\overline{x}=0$, respectively, there is

$$\overline{w}_I=e^{-Fk_1\cdot(\overline{x}+\delta_1)},\quad Fk_I=\frac{K\cdot\lambda_s}{\rho_g\cdot u_g^2\cdot C_{p,g}}\cdot e^{-\frac{E\cdot 2}{R\cdot(T_{ig}+T_0)}} \tag{14}$$

$$\Theta_I=c_3\cdot e^{k_3\cdot\overline{x}}+c_4\cdot e^{k_4\cdot\overline{x}} \tag{15}$$

$$c_3=-\frac{e^{(k_3-k_4)\cdot\delta_1}}{1-e^{(k_3-k_4)\cdot\delta_1}},\quad c_4=\frac{1}{1-e^{(k_3-k_4)\cdot\delta_1}}$$
$$k_{3,4}=\frac{\alpha}{2}\left(1\pm\sqrt{1+4\cdot\beta/\alpha^2}\right) \tag{16}$$

In the chemical reaction region the diffusion is important and Eq. (7) is integrated assuming that in the exponential term of Fk the temperature is constant and equal to the ignition temperature. As a result, there is

$$\overline{w}_{II}=e^{k_2\cdot\overline{x}-b},\quad k_{1,2}=\frac{1\pm\sqrt{1+4\cdot Le\cdot Fk}}{2\cdot Le}$$
$$b=Fk_I\cdot\delta_1,\quad Fk=\frac{K\cdot\lambda_s\cdot e^{-\frac{E}{R\cdot T_{ig}}}}{C_{p,g}\cdot\rho_g\cdot u_g^2} \tag{17}$$

Here for the general solution $\overline{w}_{II}=c_1\cdot e^{k_1\cdot\overline{x}}+c_2\cdot e^{k_2\cdot\overline{x}}$ the first term was cancelled, and $C_1=0$, because k_1 is positive. Constant C_2 was found from the boundary condition: $\overline{w}_I=\overline{w}_{II}$, $\overline{x}=0$. With regards to the combustion region, it can be said that it starts where the temperature is equal to the ignition temperature, $\Theta=1$, and ends in $\overline{x}=\delta$ where the mass fraction of combustible is nearly null: $\overline{x}\to\delta,\overline{w}\to\xi$. Therefore, from solution (17) for $\overline{w}_{II}$ it has $\xi=e^{k_2\cdot\delta-b}$ and solving for δ:

$$\delta=(\ln\xi-b)/k_2 \tag{18}$$

After having replaced the corresponding numerical values, it was found that $u_g = 0.43$ m/s, $\xi = 10^{-3}$, $Fk = 1.818$, $Le = 1.209 \times 10^{-3}$, $k_2 = -1.814$, $Fk_I = 6.258 \times 10^{-4}$, $b = 6.438 \times 10^{-2}$ and $\delta = 3.845$. The last value of δ belongs to the dimensional extension of reaction region $l = \delta \cdot X_c = 9.34$ mm. From this, it is seen that the theoretical predictions about combustion region thickness are correct because they are in agreement with the experimental observations of Aldushin et al. [5], Zhdanok et al. [14], which indicates that this value is several millimetres. Replacing in (18) the expression for k_2 and numerical values of ξ, and b, we have

$$\delta = -\frac{9.21 \cdot Le}{1 - \sqrt{1 + 4 \cdot Le \cdot Fk}} \tag{19}$$

It is known [6,20] that combustion region thickness has a direct impact over the NO_x production and therefore it would be interesting to know the functional dependence of $\delta = \delta(Le, Fk)$. Changing the dimensionless parameters $Le = \lambda_g/\lambda_s$ at range $10^{-5} \leqslant Le < 10^{-2}$, it was found that the combustion region dimensionless thickness changes slightly staying at about 2.5. However, with an increase of 1–4 in Fk, decreases from 4.6 to 1.16, are observed in the value of δ, which is desirable in practice. From the definition of Fk as a dimensionless number, it is seen that this value can increase, for example, filling physical systems with solid particles of greater thermal conductivity, λ_s, or filtrating the mixture with the least possible speed. Besides, it is also possible to decrease the combustion region thickness using mixtures with minor activation energies and higher ignition temperatures. Now, in order to integrate the energy differential equation in the second region, the derivative $\overline{w}'$ in Eq. (8) is replaced from Eq. (17) for $\overline{w}_{II}$. The resulting energy equation is non-homogeneous and is solved by means of the Lagrange method:

$$\Theta_{II} = e^{k_3 \cdot \bar{x}}(c_7 + c_7(\bar{x})) + e^{k_4 \cdot \bar{x}}(c_8 + c_8(\bar{x})) \tag{20}$$

where

$$\begin{aligned} c_7(\bar{x}) &= \frac{\gamma \cdot k_2 \cdot e^{-b}}{(k_3 - k_4) \cdot (k_2 - k_3)} \cdot e^{(k_2 - k_3) \cdot \bar{x}} \\ c_8(\bar{x}) &= \frac{\gamma \cdot k_2 \cdot e^{-b}}{(k_4 - k_3) \cdot (k_2 - k_4)} \cdot e^{(k_2 - k_4) \cdot \bar{x}} \end{aligned} \tag{21}$$

Finally in the third region $\overline{w} = \overline{w}' = 0$, and the solution to Eq. (8) with the boundary condition $\Theta = 0, \bar{x} \to \infty$ is

$$\Theta_{III} = c_{10} \cdot e^{k_4 \cdot \bar{x}} \tag{22}$$

where the constants c_7, c_8, c_{10} and the combustion wave displacement speed u_{FC} are found together from boundary conditions $\Theta_{II} = 1$ at $\bar{x} = 0$, $\Theta_{II} = \Theta_{III}$ and $\Theta'_{II} = \Theta'_{III}$ at $\bar{x} = \delta$ and $\Theta'_I = \Theta'_{II}$ at $\bar{x} = 0$, respectively:

$$\begin{aligned} &c_7 = -c_7(\delta), \quad c_8 = 1 - c_7(0) - c_8(0) + c_7(\delta) \\ &c_{10} = c_8 + c_8(\delta) \end{aligned} \tag{23}$$

$$\begin{aligned} &1 - \frac{u_{FC}}{u_t} = \sqrt{\left(\frac{C \cdot h_c \cdot w_0}{C_{p,g}(T_{ig} - T_0)}\right)^2 - \frac{4\beta \cdot \lambda_s}{(uC_p\rho)_g^2}} \\ &C = \left(e^{-b}\frac{1 - e^{(\ln \xi + b)\varsigma}}{\varsigma}\right), \quad \zeta = 1 - \frac{k_3}{k_2}, \quad u_t = \frac{(\rho \cdot c_p)_g \cdot u_g}{(\rho \cdot c_p)_S} \end{aligned} \tag{24}$$

It is seen from formula (24) that unlike the solution of Foutko et al. [12], the combustion wave displacement speed depends on the characteristic parameters of both heat transport and mass. However, when $k_3/k_2 \to 0$, $\varsigma \to 1$, and $C \to 1$ implies that formula (24) fully coincides with result of Foutko et al. [12].

To obtain the analytical prediction for the dimensionless combustion wave velocity u_{FC} from Eq. (24), the ignition temperature T_{ig} must be defined. The former may be estimated by considering the temperature evolution of a gas element moving through the heated porous media in a self-sustaining combustion wave. Assuming a moderate interfacial heat exchange coefficient and that the reaction (fuel consumption) is almost completed for the distance of the order of the porous size, the temperature increase for the gas element may be treated as a thermal explosion. Thus, one can directly apply the corresponding theory of Frank–Kamenetzki [21] with the correction for fuel consumption, and derive an approximation for the ignition temperature T_{ig} in the following implicit form:

$$\frac{e^{\frac{E}{R \cdot T_{ig}} - 1}}{(E/R \cdot T_{ig})^{7/3}} = \left(\frac{h_c^2 \cdot w_0^2 \cdot R^5}{2 \cdot \pi^2 \cdot C_{p,g}^2 \cdot E^2}\right) \cdot \left(\frac{d_p \cdot \varepsilon \cdot T_0 \cdot K}{u_g}\right) \tag{25}$$

4. Results and discussion

To support and to test the analytical solutions obtained here, a computational procedure was performed using a full set of the basic equations. These were calculated using the implicit finite difference discretization and the Tree-Diagonal Matrix Algorithm. The time step was 0.01 s and 800 grid points were used for numerical calculations. In Fig. 3 the analytical solution obtained in this paper is compared to the numerical solution built in the same paper. Many coincidences between the compared cases are observed in the figures. Apart from that, the maximum

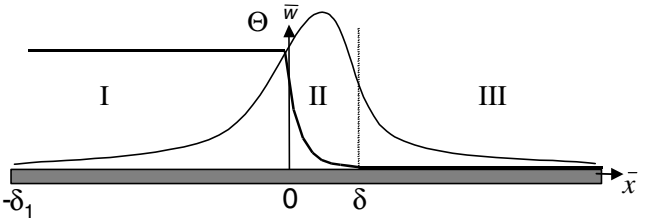

Fig. 2. Three representative regions of the system according to the working hypothesis.

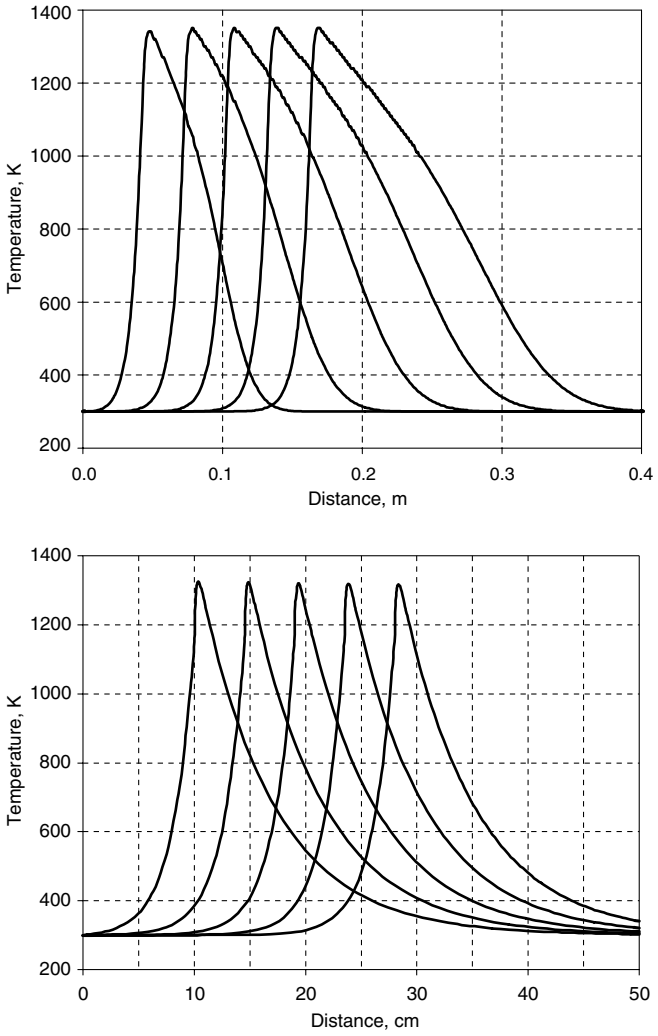

Fig. 3. Stable combustion wave propagation for $u_{FC} = 1.7 \times 10^{-4}$ m/s, $\varphi = 4.88$, $u_g = 0.43$ m/s, $T_{ig} = 1150$ K, time interval 5 min between adjacent peaks: top—numerical solution, bottom—analytical solution.

temperatures reached at the combustion front according to our solution are found within the maximum temperature values in the numerical solution. Furthermore, it was found that the analytical solution was very sensitive to the value of the wave displacement speed. Consequently, the exact calculation of its value is fundamental for the construction of the correct solution.

Finally, the constructed analytic solution is analyzed in terms of five dimensionless parameters which define heat and mass transport in mathematical model (7), (8): α, β, γ, Le and Fk. The first parameter, α, is related with the movement speed of the combustion front. Considering that at each test, front speed is permanent and positive, its variation range is $0.01 \leqslant \alpha \leqslant 0.1$. Moreover, $\beta = 4.5505 \times 10^{-3}$, $\gamma = 5.2636 \times 10^{-1}$, $Le = 1.209 \times 10^{-3}$, and $Fk = 1.6091$. From Fig. 4 it can be seen that with an increase in value of α from 0.01 to 0.1 local temperatures decrease in the pre-heating region (region I in Fig. 2). At this region, the heating of the solid media is achieved by conduction of

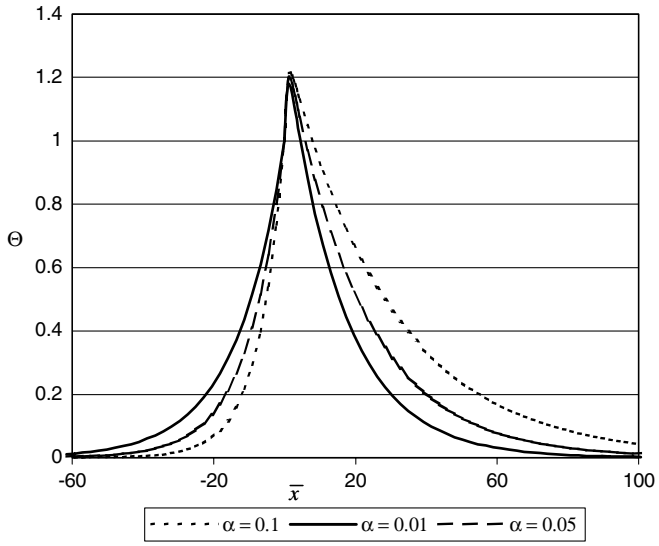

Fig. 4. Variation of the dimensionless temperature with the parameter α.

the heat coming from the combustion front and cooling occurs by forced convection of the entrance gas. Thus, increasing the values of parameter α increases gas speed, the porous matrix at that region is therefore cooled and, as result, lower temperatures are obtained. At the combustion region (region II at Fig. 2) the highest temperature also increases with an increase in parameter α. These changes also agree with physical reality because the forced convection becomes more intensive each time and consequently, brings to the combustion front a larger amount of absorbed enthalpy from the porous matrix at the pre-heating region. Finally, in the post-combustion region, local temperatures get higher when parameter α increases due to forced convection, which brings heat to regions each time further away from the combustion zone within the porous media.

The second dimensionless parameter, β, is related with heat losses from the system through lateral surfaces as natural convection and thermal radiation. Its variation range is next considered: $0.0 \leqslant \beta \leqslant 0.01$. Moreover, $\alpha = 8.7916 \times 10^{-2}$, $\gamma = 5.2636 \times 10^{-1}$, $Le = 1.209 \times 10^{-3}$, and $Fk = 1.6091$. The case $\beta = 0$ corresponds to a system in thermal isolation from the environment. From Fig. 5 it can be seen that at the pre-heating region, local temperatures are not very high and, consequently, heat flow from the system to the environment (proportional to $(T - T_0)$) is weak. Therefore, varying β shows very little temperature fluctuations. However, at the combustion region and beyond, where combustion products are found at high temperatures, considerable changes are seen from local temperatures.

The third dimensionless parameter in energy equation (8), γ, is related to the combustion enthalpy, h_c. Considering the physical properties of solids and gasses defined previously, its value is: $\gamma = 0.555$. Consequently, γ is varied in the range $0.25 \leqslant \gamma \leqslant 5.0$, keeping constant the rest of the parameters: $\alpha = 8.7916 \times 10^{-2}$, $\beta = 4.5505 \times 10^{-3}$, $Le =$

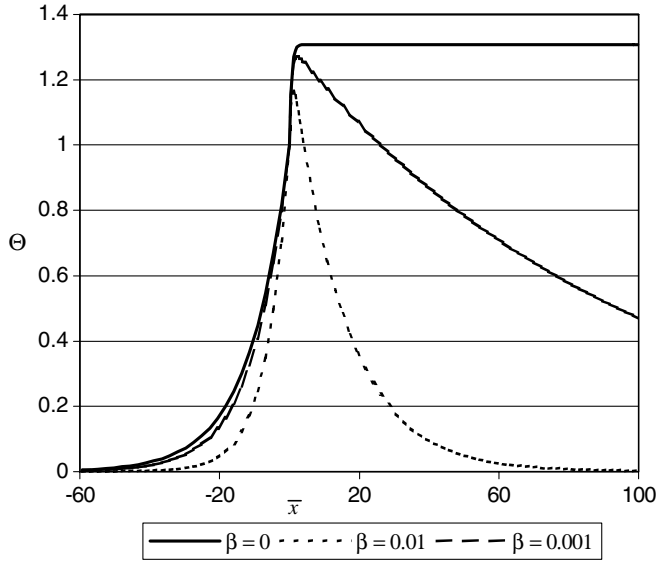

Fig. 5. Variation of the dimensionless temperature with the parameter β.

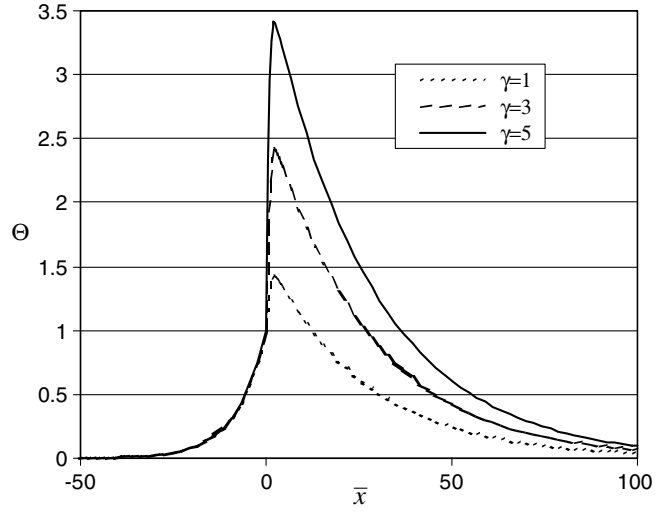

Fig. 6. Variation of the dimensionless temperature with the parameter γ.

1.209×10^{-3}, $Fk = 1.6091$. From Fig. 6 it can be seen that with an increase in the value of γ, the temperatures increase both at the combustion region, and in the region after the front. However, at the pre-heating region were the chemical reaction practically does not occur, there is practically no change in temperature for different values of γ.

From the mathematical model (7), (8) it is seen that parameters α, β and γ, all influence temperature and do not affect the distribution of combustible mass fraction across the system. However, the parameters Le and Fk do influence combustible mass fraction distribution and, consequently, the impact of their values on temperature and fuel mass fraction profiles of the system is analyzed in Figs. 7 and 8.

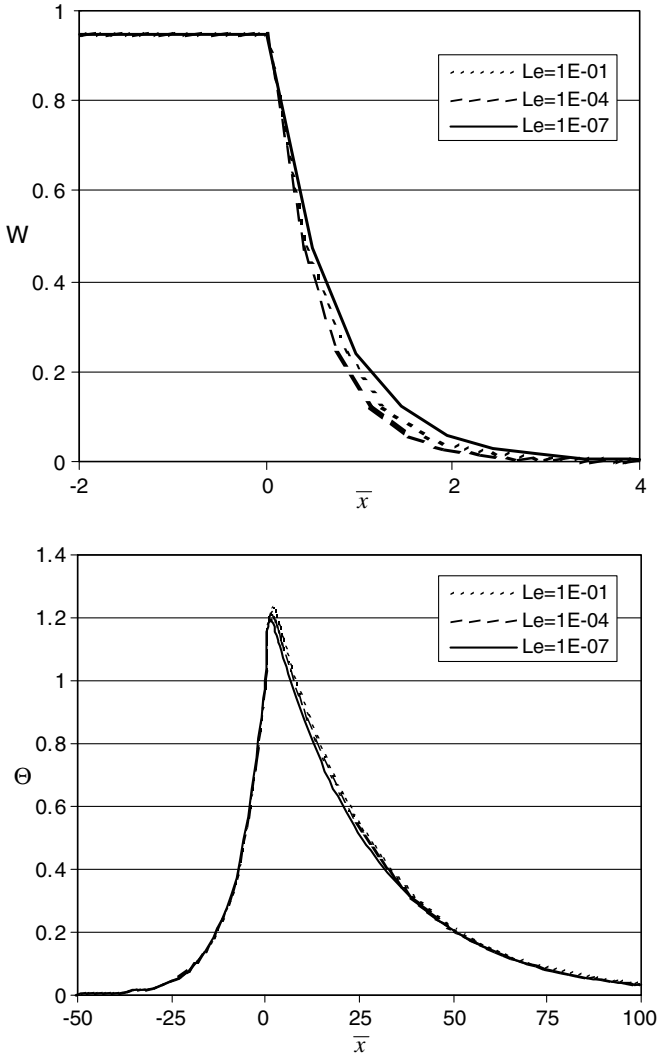

Fig. 7. Variation of the mass fraction of the fuel and of the temperature with the parameter Le.

The fourth-dimensionless parameter of the methane mass equation (7), Le, is related with mass transport by diffusion, which is considerable only at the combustion region. Therefore, its variation is in the range $10^{-7} \leqslant Le \leqslant 10^{-1}$ with values of $\alpha = 8.7916 \times 10^{-2}$, $\beta = 4.5505 \times 10^{-3}$, $\gamma = 5.2636 \times 10^{-1}$, and $Fk = 1.6091$. From Fig. 7 it can be seen that variations in Le affect both temperature and mass fraction of fuel distribution across the system.

However, the fifth parameter, Fk, which is related with fuel consumption by chemical reaction, considerably affects the fuel mass fraction and temperature profiles. Its value varies in the range $0.8265 \leqslant Fk \leqslant 2.429$ keeping constant the following parameters: $\alpha = 8.7916 \times 10^{-2}$, $\beta = 4.5505 \times 10^{-3}$, $\gamma = 5.2636 \times 10^{-1}$, and $Le = 1.209 \times 10^{-3}$. The results of this analysis are shown in Fig. 8, where it can be seen that the major variations of both fuel mass fraction and temperatures happen at the combustion region.

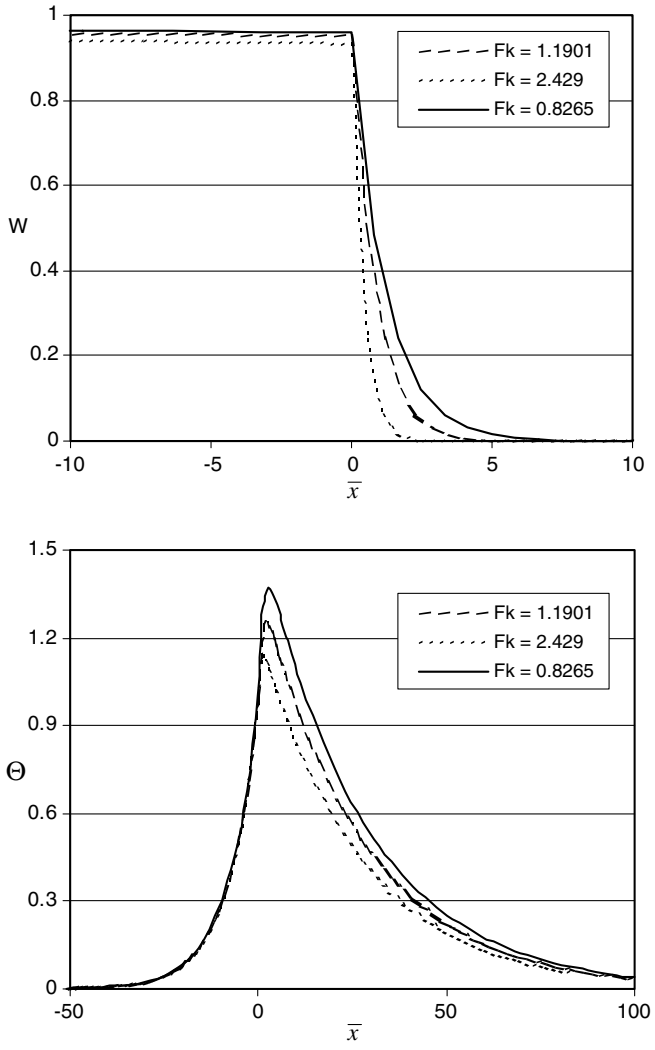

Fig. 8. Variation of the fuel mass fraction and of the temperature with the parameter *Fk*.

5. Conclusions

The theoretical study of gas combustion in inert porous media concluded with the analytical construction of a series of simple algebraic formulas, which represents the methane mass fraction and temperature profiles in a semi-infinite inert porous media. Also, simple formulas were built, which allow the prediction of the combustion wave velocity in the system and the thickness of the reaction region. Five dimensionless parameters define heat and mass transport in the mathematical model. The analytical solution satisfactorily coincides with numerical and analytical solutions of other works. In the study it is shown that the reaction region is not infinitely thin, as is usually assumed. An analytical formula is developed which predicts the thickness of the combustion region. Consequently, the results of this research can be used in the analysis of combustion waves in porous media for technical applications.

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